Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

May 2023

Mathematics: applications and interpretation

Higher level

Paper 3

16 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

R Marks awarded for clear **Reasoning**.

AG Answer given in the question and so no marks are awarded.

FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme eg ***M1***, ***A2***.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.

- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.

- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.

- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.

- Where the markscheme specifies ***A3***, ***M2*** *etc.*, do **not** split the marks, unless there is a note.

- The response to a "show that" question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.

- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.

- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part. Examples:

	Correct answer seen	Further working seen	Any FT issues?	Action
1.	$8\sqrt{2}$	5.65685... *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	0.468111… (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g. *(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.
- If the candidate’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a “show that” question correctly, and then in subsequent parts use their incorrect answer rather than the given value.
- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.
- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was “Hence”.

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.
- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and $1{,}9$ or 1000 and $1{,}000$ and 1.000.
- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures*.

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer to a "correct" level of accuracy (e.g 3 sf) in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

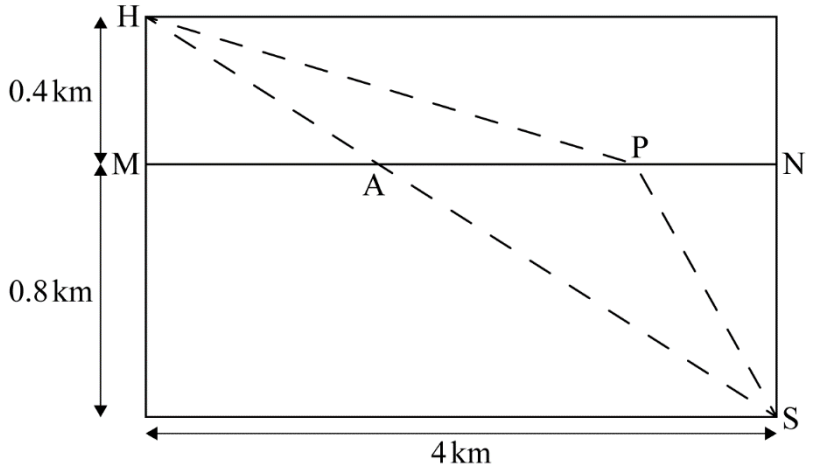
A GDC is required for this paper, but If you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is “first”.

1. (a)



METHOD 1

$\hat{\text{MHS}} = \left(\tan^{-1} \frac{4}{1.2} = \right) \ 73.3007...^\circ \ \text{OR} \ 1.27933...$ ***(A1)***

use of trigonometry to find HA or AS ***(M1)***

$\text{HA} = \frac{0.4}{\cos \hat{\text{MHS}}}$ **AND** $\text{AS} = \frac{0.8}{\cos \hat{\text{MHS}}}$ ***(A1)***

$(\text{HA} = 1.39204...$ and $\text{AS} = 2.78408...)$

use of $\text{time} = \frac{\text{distance}}{\text{speed}}$ for either of their distances ***(M1)***

time taken $= \left(\frac{\text{AH}}{15} + \frac{\text{AS}}{5} \right)$

$0.649618...$ (hours) ***(A1)***

(38.97712… minutes)

therefore 39 (mins) ***A1FT***

Note: Allow ***FT***, within the question part, from their time in hours for the final ***A1***.

METHOD 2

EITHER

use of similar triangles to identify either length MA or AN ***(M1)***

$\left(\frac{4}{3} \text{ or } \frac{8}{3}\right)$

attempt to use Pythagoras for either triangle AMH or ANS ***(M1)***

$\mathrm{AH}^2 = 0.4^2 + \left(\frac{4}{3}\right)^2$ **AND** $\mathrm{AS}^2 = 0.8^2 + \left(\frac{8}{3}\right)^2$ ***(A1)***

OR

attempt to use Pythagoras for larger triangle ***(M1)***

$\mathrm{SH}^2 = 4^2 + 1.2^2$

$\mathrm{AH} = \frac{1}{3}\sqrt{4^2 + 1.2^2}$ **AND** $\mathrm{AS} = \frac{2}{3}\sqrt{4^2 + 1.2^2}$ ***(M1)(A1)***

THEN

$(\mathrm{HA} = 1.39204...$ and $\mathrm{AS} = 2.78408...)$

use of $\text{time} = \frac{\text{distance}}{\text{speed}}$ for either of THEIR distances ***(M1)***

time taken $= \left(\frac{\mathrm{AH}}{15} + \frac{\mathrm{AS}}{5}\right)$

$0.649618...$ (hours) ***(A1)***

(38.97712… minutes)

therefore 39 (mins) ***A1FT***

Note: Allow ***FT***, within the question part, from their time in hours for the final ***A1***.

[6 marks]

(b) (i) $\text{PH}^2 = 0.4^2 + x^2$ **AND** $\text{PS}^2 = 0.8^2 + (4-x)^2$ ***A1***

Note: This ***A1*** can be implied by a clear expression for the time in each region coming from distance / speed below.

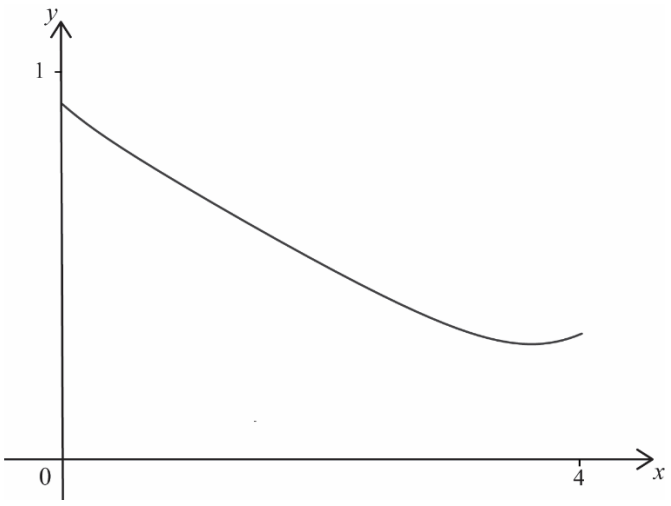
$$T(x) = \frac{\text{PH}}{15} + \frac{\text{PS}}{5}$$ ***(M1)***

$$T(x) = \frac{\sqrt{0.4^2 + x^2}}{15} + \frac{\sqrt{0.8^2 + (4-x)^2}}{5}$$ ***A1***

$$T(x) = \frac{\sqrt{0.4^2 + x^2} + 3\sqrt{0.8^2 + (4-x)^2}}{15}$$ ***AG***

[3 marks]

(ii)



correct shape with minimum point nearer $x = 4$ than $x = 0$ ***A1***
correct (approximate) y-intercept, $0.843\ldots$ (must be clearly below 1) ***A1***

[2 marks]

(iii) using the GDC, at the minimum $x = 3.72$ $(3.71898...)$ ***A1***

Note: Do not accept coordinates of the minimum point.

[1 mark]

(iv) finding their $T(x)$ for their value of x ***M1***
$T(x) = 0.418946\ldots$
so time saved $(= 38.97712\ldots - 25.1367\ldots \text{mins})$ = 14 (mins) ***A1***

[2 marks]

(c) (i) attempt at chain rule ***M1***

$$T'(x) = \frac{1}{15}\left(\frac{x}{\sqrt{0.4^2 + x^2}} - \frac{3(4-x)}{\sqrt{0.8^2 + (4-x)^2}}\right)$$ ***A1A1***

Note: Award ***A1*** for each correct term. Accept any equivalent form i.e. condone fractions not simplified.

[3 marks]

(ii) setting their $T'(x) = 0$ ***M1***

Note: This requires more than just a statement that the derivative equals zero – they must use their attempt at $T'(x)$.

$$\frac{1}{15}\left(\frac{x}{\sqrt{0.4^2 + x^2}} - \frac{3(4-x)}{\sqrt{0.8^2 + (4-x)^2}}\right) = 0$$

$$\frac{x}{\sqrt{0.16 + x^2}} = \frac{3(4-x)}{\sqrt{0.64 + (4-x)^2}}$$ ***AG***

[1 mark]

(iii) **METHOD 1**

$\cos \hat{\text{HPM}} = \frac{x}{\sqrt{0.16 + x^2}}$ **AND** $\cos \hat{\text{SPN}} = \frac{4-x}{\sqrt{0.64 + (4-x)^2}}$ ***A1***

substituting in the above equation and rearranging ***M1***

$\cos \hat{\text{HPM}} = 3\cos \hat{\text{SPN}}$ leading to $\frac{\cos \hat{\text{HPM}}}{\cos \hat{\text{SPN}}} = 3 = \left(\frac{15}{5}\right)$

verifying the result ***AG***

METHOD 2

$$\frac{x}{\sqrt{0.16 + x^2}} = \frac{3(4-x)}{\sqrt{0.64 + (4-x)^2}}$$

attempt to rearrange into a quotient ***M1***

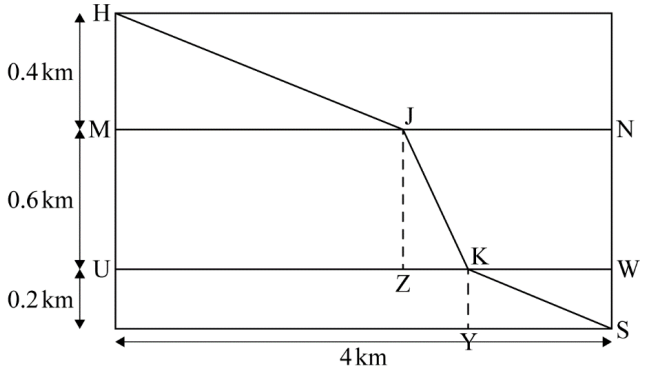
$$\left(\frac{15}{5} = 3 =\right) \frac{\frac{x}{\sqrt{0.16 + x^2}}}{\frac{4-x}{\sqrt{0.64 + (4-x)^2}}}$$

$$= \frac{\cos \hat{\text{HPM}}}{\cos \hat{\text{SPN}}}$$ ***A1***

verifying the result ***AG***

[2 marks]

(d) **METHOD 1**



let $\mathrm{MJ} = y$ km and W and Z be the points on the new boundary directly below N and J

attempt to find ZK in terms of MJ ***M1***

$(\mathrm{KW} = 0.5y)$

$\mathrm{ZK} = (4 - 1.5y)$ km. ***A1***

attempt to use the result from (c)(iii) at J ***M1***

$$\frac{\cos \mathrm{H\hat{J}M}}{\cos \mathrm{Z\hat{K}J}} = \frac{y}{\sqrt{y^2 + 0.4^2}} \div \frac{(4-1.5y)}{\sqrt{(4-1.5y)^2 + 0.6^2}} = \frac{15}{5}$$ ***A1***

Note: Accept $\cos \mathrm{N\hat{J}K}$ in place of $\cos \mathrm{Z\hat{K}J}$.

$$\left(\text{leading to } \frac{y}{\sqrt{y^2 + 0.16}} = \frac{3(4-1.5y)}{\sqrt{(4-1.5y)^2 + 0.36}} \right)$$

valid method for solving this equation, eg drawing graphs of both sides of the equation, using SOLVER, etc. ***(M1)***

solution is $y = 2.53$ ***A1***

METHOD 2

combining the field into one region with height 0.6 km ***M1***

$$\cos \mathrm{H\hat{P}M} = \frac{x}{\sqrt{0.36 + x^2}}$$

$$\cos \mathrm{S\hat{P}N} = \frac{4-x}{\sqrt{0.36 + (4-x)^2}}$$ ***A1***

Note: Both expressions, or their ratio, are required for the ***A1*** to be awarded.

therefore

$$\frac{x\sqrt{0.36 + (4-x)^2}}{(4-x)\sqrt{0.36 + x^2}} = 3$$ ***A1***

valid method for solving ***(M1)***

attempting to find MJ in terms of x e.g. $\mathrm{MJ} = \frac{2}{3}x$ ***M1***

so $\mathrm{MJ} = 2.53$ ***A1***

[6 marks]

Total [26 marks]

2. (a) (i) because the (population) standard deviation(s) are unknown ***A1***

Note: Ignore any references to sample size.

[1 mark]

(ii) **EITHER**
he has no idea beforehand which way the difference would be if there is a difference ***A1***

OR
he is only interested that there is a difference (not the direction) ***A1***

[1 mark]

(b) (i) **EITHER**
$H_0: \mu_F = \mu_G;\ H_1: \mu_F \neq \mu_G$ ***A1***

OR
$H_0: \mu_D = 0;\ H_1: \mu_D \neq 0$ ***A1***

Note: Accept an equivalent statement in words, must include mean and reference to "population mean" / "mean for all those taking the French exam" etc. for the first ***A1*** to be awarded. The terms "on average" and "generally" are also acceptable to indicate populations. Do not accept an imprecise "the means are equal".

Do not accept "There is (no) (significant) evidence of a difference between μ_F and μ_G" for either hypothesis or "There is (no) significant difference between marks in French and German".

[1 mark]

(ii) Generate a third column giving French mark – German mark or German mark – French mark. ***(M1)***
e.g.

French mark	German mark	Difference
42	39	3
65	66	–1
82	71	11
…	…	…

p-value $= 0.153$. ***A1***

[2 marks]

(iii) The p-value gives the probability of seeing the observed difference in means (or a larger difference) assuming H_0 to be true. ***A1***

Note: Do not accept "the probability that the data occurs by chance" or similar.

[1 mark]

(iv) because $0.153 > 0.05$ ***R1***
EITHER
there is not (significant evidence of) a difference between the (population) means ***A1***
OR
fail to reject H_0 (accept "accept H_0") ***A1***

Note: Do not award ***R0A1***.
Remember to ***FT*** from part (b)(ii).
Do not award the final ***A1*** if the null hypothesis in part (b)(i) is logically wrong (i.e. if the null and the alternative have been reversed or are nonsense) but this can be awarded if part (b)(i) is just poorly communicated.

[2 marks]

(c) (i) $\mathrm{H}_0 : \rho = 0 ;\ \mathrm{H}_1 : \rho > 0$ ***A1***

Note: Condone $\mathrm{H}_0 : \rho \leq 0$.

[1 mark]

(ii) p-value $= 0.00286$ ***A2***
$0.00286 < 0.05$ ***R1***
he should conclude that the two sets of marks are (generally) positively correlated ***A1***

Note: Allow ***FT*** from any test for correlation.
Do not award ***R0A1***.
The final ***R1A1*** should follow through from their p-value.
Do not award the final ***A1*** if the null hypothesis in part (c)(i) is wrong (i.e. if the null and the alternative have been reversed or are nonsense), but this can be awarded if part (c)(i) is just poorly communicated.
The final conclusion must be in context.

[4 marks]

(d) (i) the regression line of German on French is
$\text{German} = 10.2393... + 0.737495... \text{French}$ ***(A1)***
EITHER
substituting $\text{French} = 58$ into their regression line ***(M1)***
OR
sketch showing regression line and $x = 58$ ***(M1)***

THEN
Paul's German mark $= 53$ ***A1***

Note: Accept an answer of 53.0 $(53.0140...)$ or 52.9 as integer results are not explicitly stated in the question.
Regression lines may be written in terms of y and x.

[3 marks]

(ii) recognizing need to use line French on German
$\text{French} = 4.04116... + 1.01122...\text{German}$ ***(A1)***
putting $\text{French} = 71$, Sue's German mark $= 66$ ***A1***

Note: Accept an answer of 66.2 $(66.2158...)$ or 66.3 as integer results are not explicitly stated in the question.
Although not required in the markscheme as presented, candidates may have considered $\text{French} = 70.5$ and $\text{French} = 71.5$; this is valid and will lead to the correct answer.
If the line German on French is used in part (d)(ii) the answer is 63; award ***A0A0***.

[2 marks]

(e) (i) **EITHER**
the maximum value of τ occurs when all pairs are concordant so max $= +1$
the minimum value of τ occurs when all pairs are discordant so min $= -1$
A1

OR

when all concordant $C - D = \frac{n(n-1)}{2}$, and when all discordant $C - D = -\frac{n(n-1)}{2}$
A1

OR

when all concordant $C = \frac{n(n-1)}{2}$, $D = 0$ and when all discordant $C = 0$, $D = \frac{n(n-1)}{2}$
A1

THEN
hence the range is $[-1, +1]$ ***AG***

Note: Accept an answer which is just based on $n = 6$.

[1 mark]

(ii) $(53 - 76)(41 - 70) > 0$ ***A1***
Hence concordant ***AG***

[1 mark]

(iii) Evidence of a valid method, eg ***M1***
P_1: C, D, C, C, C
P_2 : C, C, C, D
P_3 : D, C, D
P_4 : D, C
P_5 : D
P_6 :

Note: At least one pair beyond (P_1, P_2) needs to be compared to award ***M1***.

any evidence (a statement or a list) that 15 pairs need to be considered ***A1***

$C = 9,\ D = 6$ ***A1***

using their stated C and D values in given formula with $n = 6$ ***M1***
$\frac{2(9-6)}{6(6-1)}$ **OR** $\frac{9-6}{15}$
$\tau = 0.2$ ***AG***

[4 marks]

(f) (i) H_0: There is no (underlying) association (or correlation) between the two sets of marks
H_1: There is an (underlying) association (or correlation) between the two sets of marks ***A1***

Note: Do not accept independence in the hypotheses.

[1 mark]

(ii) τ does not lie in the critical region **OR** $0.2<0.733$ ***R1***
EITHER
there is insufficient evidence to indicate that there is an association between the two sets of marks ***A1***

OR
fail to reject H_0 (accept "accept H_0") ***A1***

Note: Do not award ***R0A1***.
In this question the final ***A1*** mark can be awarded for "fail to reject H_0" or "accept H_0" even if the hypotheses in (f)(i) are the wrong way round as the critical region is given.

[2 marks]

(g) no ***A1***
because scaling the marks will not affect the concordances/ discordances ***R1***

Note: Do not award ***A1R0***.

[2 marks]
Total [29 marks]